

## **Application of Forward Kinematic Methods and Jacobi Matrix in Analyzing The Singular Conditions of The Kawasaki Industrial Series Robots**

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### **Abstract**

In designing a manipulator, several things need to be considered, which is like the singular condition of the manipulator. This is so that the manipulators can operate in a safe work area. This research will begin with modeling 3D serial manipulator 6R using Inventor software. This 3D model is needed to get joint parameter values, namely  $a$ ,  $d$ ,  $\theta$  and  $\alpha$ . After obtaining the joint parameters, a Forward Kinematic calculation is performed to get the end effector position when given a certain input angle using Matlab software. Then variations are made on the input angle  $\theta_2$  ( $30^\circ$  to  $180^\circ$ ) and  $\theta_3$  ( $-90^\circ$  to  $60^\circ$ ) and a graph that shows working spaces obtained. To ensure that the calculations performed are correct, validation using inventor is done by providing an angle value for the second and third joints. Then the Jacobian matrix was built to obtain the singularity value of each configuration by determining the Jacobian matrix. The configuration is stated to be in a singular position if the  $\det(J)$  approaches 0. With the variation in input angle obtained, the manipulator range is  $-471$  mm to  $2967$  mm on the Z axis and  $-1936$  mm to  $2699$  mm on the X axis. The singular manipulator is occur when  $\theta_3 = -78.15^\circ$ . From these results it can be stated that the manipulator cannot be in the configuration of  $\theta_3 = -78.15^\circ$ . This is in line with the manufacturer's recommendations which only allow  $\theta_3$  minimum is  $-75^\circ$ .

**Keywords:** arm robot, DH parameter, forward kinematics, serial manipulator, singularity.

### **1. Introduction**

Banjar Currently, many industries use automation systems, such as robotic manipulators [1]. Manipulator robot is a multifunctional robot that can be reprogrammed and designed to move materials, parts, tools or a special device with programmable movement variables to perform special jobs [2]. Robots can be divided into four types, namely mobile robots, robot manipulators, a combination of mobile robots and manipulator robots and humanoid robots. While the manipulator itself consists of a series and parallel manipulator.

The series manipulator consists of 5 types, namely: cartesian, cylindrical, spherical, SCARA and artificial manipulator [3]. This research will focus on artificial serial manipulator.

The things that must be considered in the design of a manipulator include the accuracy and range of the manipulator, which is known as the singular condition. Wahyudi's previous research on Trajectory Planning Parallel Manipulator 3R has been carried out by Wahyudi [4]. As for the serial manipulator on the graphical representation of the work area,

Djuric has also done, Singularity Manipulator PUMA 6 DOF by Cheng and Posture Optimization of Robot 6R by Lin [5], [6], [7]. In this research, the Working Space Serial Manipulator Type 6R will be carried out. The method used in this research is the forward kinematic method. Forward Kinematic is a method in robot kinematics analysis by knowing the input in each joint space so that the coordinates and orientation of the end effector can be known [8]. In designing a manipulator robot, it is necessary to pay attention to the work area that will be passed by the manipulator. Working space is the entire area that can be reached by the end effector of a manipulator within a predetermined singularity limit. While singularity in general is a condition where the mechanism or kinematic chain loses one or more degrees of freedom.

This research begins by determining the DH parameters on the manipulator to be tested by building a Jacobi Matrix for each desired manipulator configuration. If the value of the determinant of the Jacobi Matrix is close to null, then the configuration is close to the singular position. Conversely, if the value is close to 1 then the configuration is in the ideal position.

## 2. Method

This research begins with 3D manipulator modeling with the Kawasaki CX 210L Type CX 210L Basic Robot Manipulator. The object of this research is a 3D mode obtained from the industrial robot manufacturer Kawasaki Robotics through the company's website (Figure 1).

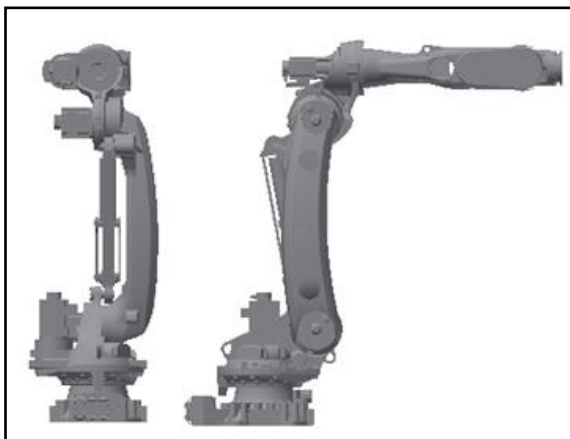


Figure 1. Research object

### A. DH Parameters

Material DH Parameter is a method used to determine the displacement position of a mechanism to its initial position by

transforming the coordinates between two adjacent links.

To collect research data in the form of DH Parameters, it is done graphically by measuring the dimensions of the 3D model of the Kawasaki Robotics industrial robot using Inventor software. After simplification can be seen the kinematic chain mechanism as shown in Figure 2.

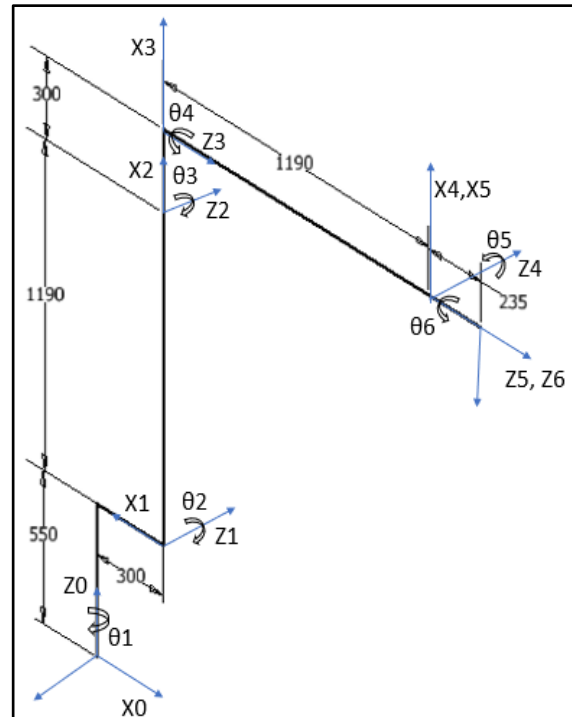


Figure 2. Kinematic chain manipulator 6R

The value of DH Parameters for each mechanism can be seen in Table 1.

Table 1. DH parameters

Join	a (mm)	d (mm)	$\theta$	A
0-1	300	550	$\theta_1$	$-90^\circ$
1-2	1190	0	$\theta_2 (90)$	$0^\circ$
2-3	300	0	$\theta_3$	$90^\circ$
3-4	0	1190	$\theta_4$	$90^\circ$
4-5	0	0	$\theta_5$	$-90^\circ$
5-6	0	235	$\theta_6$	$0^\circ$

### B. Working Space with Transformation Matrix Method

After obtaining the DH parameter, then forward kinematic calculations are carried out to obtain the end effector position for each configuration. In equation 1, it can be seen that the standard equation states the relationship between a joint point and the next joint point in a kinematic chain. This matrix is called the homogeneous transformation matrix.

$${}^{i-1}T_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where, the red box is the orientation coordinate and the blue box is the final position. To get the position of the end effector ( $X_1, Y_1, Z_1$ ) relative to the reference point ( $X_0, Y_0, Z_0$ ), it can be done by determining the homogeneous transformation matrix for each joint space and then multiplying all the matrix obtained. In this study, according to the research object that has 6 degrees of freedom, 6 homogeneous transformation matrix are obtained, namely,  ${}^0_1T, {}^1_2T, {}^2_3T, {}^3_4T, {}^4_5T, {}^5_6T$ . The final position of the end effector is the 4x4 matrix shown in equation 2, where the 3x3 matrix (blue box) represents the orientation direction of the end effector and the 3x1 matrix (red box) represents the end effector's final position.

$${}^0_6T_i = \begin{bmatrix} R11 & R12 & R13 & R14 \\ R21 & R22 & R23 & R24 \\ R31 & R32 & R33 & R34 \\ R41 & R42 & R43 & R44 \end{bmatrix} \quad (2)$$

**C. Singularity Degree Using the Jacobi Matrix Method**

After The singularity degree test uses the Jacobi Matrix method, which starts by determining the sample point in the manipulator working space area that has been obtained. After that, a Jacobi Matrix is built for each point that has been determined, then each Jacobi Matrix is determined to find out the singularity value of each point as in equation 3.

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \dots & \frac{\partial f_1}{\partial q_n} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \dots & \frac{\partial f_2}{\partial q_n} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \dots & \frac{\partial f_3}{\partial q_n} \\ \frac{\partial f_4}{\partial q_1} & \frac{\partial f_4}{\partial q_2} & \dots & \frac{\partial f_4}{\partial q_n} \\ \frac{\partial f_5}{\partial q_1} & \frac{\partial f_5}{\partial q_2} & \dots & \frac{\partial f_5}{\partial q_n} \\ \frac{\partial f_6}{\partial q_1} & \frac{\partial f_6}{\partial q_2} & \dots & \frac{\partial f_6}{\partial q_n} \end{bmatrix} \quad (3)$$

**3. Results and Discussion**

**A. Work Space**

After obtaining the transformation matrix for each joint space, nominal calculations can be carried out using the Matlab application. Figure 3 shows the calculation process which starts by defining the variables and coefficients to be used, inputting the homogeneous

transformation matrix and then giving the multiplication command.

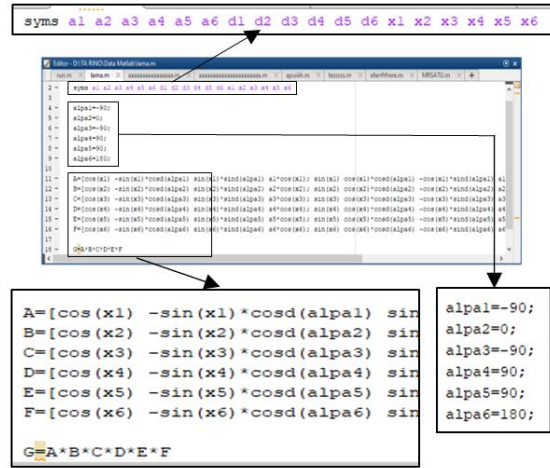


Figure 3. Transformation matrix

From these calculations, a transformation matrix (4x4) is obtained which shows the relationship between the end effector ( $X_1, Y_1, Z_1$ ) and the reference point ( $X_0, Y_0, Z_0$ ) as in equation 2. Furthermore, to get a working space graph, it can be done by varying the input angle 2 from  $30^\circ$  to  $180^\circ$  and 3 from  $-60^\circ$  to  $70^\circ$ . With these variations, a working space graph is obtained as shown in Figure 4.

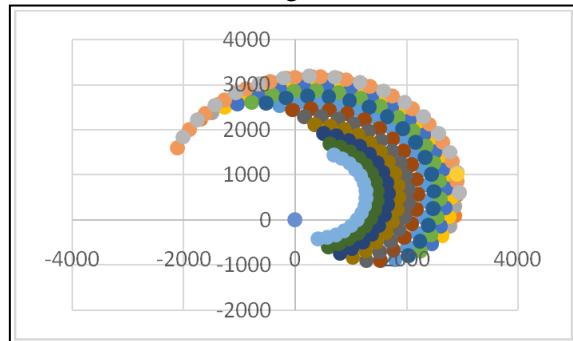


Figure 4. Working space

**B. Kinematic Forward Calculation Validation**

Furthermore, testing is carried out for several points that have been calculated using Matlab software, namely for the first position ( $X=2114.7, Y=0, Z=1597.7$ ) the second position ( $X=-2015, Y=0, Z=1831.2$ ) and the third position ( $X=-1876.4, Y=0, Z=2043.8$ ). Validation is carried out to ensure that the calculations performed are correct. The validation process will be done graphically using the Inventor software.

*First place*

For the first position the input angle given to each joint space is;  $1=0^\circ, 2=30^\circ, 3=-90^\circ, 4=0^\circ, 5=0^\circ$  and  $6=0^\circ$ . The position obtained can be seen in Figure 5.

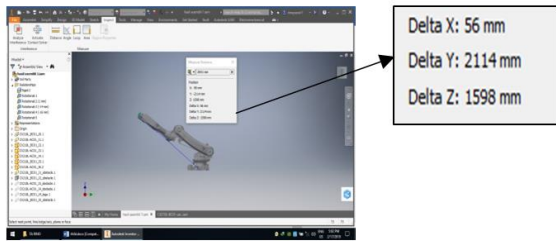


Figure 5. First place

Based on Figure 5, the coordinates obtained are,  $X = 56 \text{ mm}$ ,  $Y = 2114 \text{ mm}$ ,  $Z = 1598 \text{ mm}$ .

*Second place*

For the second position with conditions  $1=0^\circ$ ,  $2=30^\circ$ ,  $3=-80^\circ$ ,  $4=0^\circ$ ,  $5=0^\circ$  and  $6=0^\circ$ . The position obtained is shown in Figure 6.

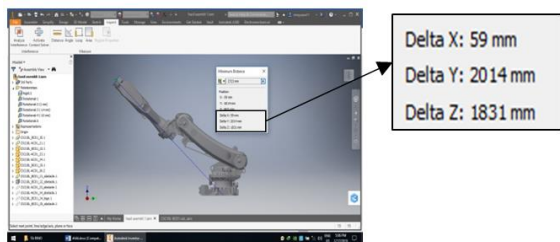


Figure 6. Second place

Based on Figure 6, the coordinates obtained are,  $X=59 \text{ mm}$ ,  $Y=2014 \text{ mm}$ ,  $Z=1831 \text{ mm}$ .

*Third place*

For the third place with conditions  $1=0^\circ$ ,  $2=30^\circ$ ,  $3=-70^\circ$ ,  $4=0^\circ$ ,  $5=0^\circ$  and  $6=0^\circ$ . The position obtained can be seen in Figure 7.

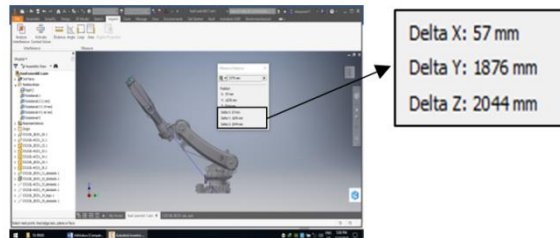


Figure 7. Third place

Based on Figure 7, the coordinates obtained are,  $X=57 \text{ mm}$ ,  $Y=1876 \text{ mm}$ ,  $Z=2044 \text{ mm}$ .

After doing these calculations, comparisons can be made with calculations using Matlab. Table 2 shows a comparison of the results of the calculations that have been carried out.

Table 2. Comparison of calculation results

Variasi Sudut ( $^\circ$ )		Hasil Perhitungan dengan Matlab (mm)			Hasil Perhitungan dengan Inventor		
$\theta_2$	$\theta_3$	X	Y	Z	X	Y	Z
0	-90	-2114.7	0	1597.7	2114	56	1598
30	-80	-2015	0	1831.2	2014	59	1831
30	-70	-1876.4	0	2043.8	1876	57	2044

Based on Table 2, the calculations carried out with Matlab and Inventor have identical

results, so it can be stated that the forward kinematic calculations that have been carried out are correct.

**C. Singularity Result**

From the points obtained in Figure 4, several samples will be taken to see the tendency of the degree of singularity along the working space area. The samples taken consisted of 6 lines drawn from the innermost to the outermost part of the working space area as shown in Figure 8.

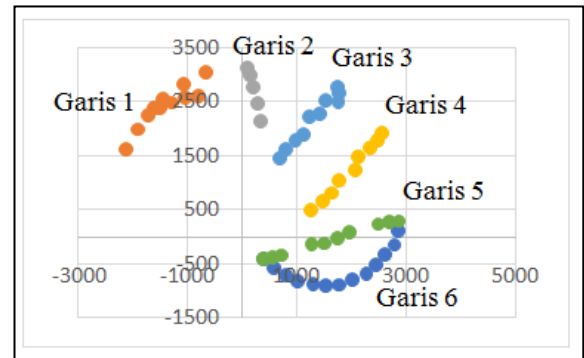


Figure 8. Singularity test

Each of these points will be tested using the Jacobi matrix method. If the determination value of the Jacobi matrix is close to 0 then the mechanism is said to be in a singular configuration. Meanwhile, to build the Jacobi matrix, it can be seen in equations 2 and 3.

Furthermore, to simplify the calculation process, simplification of the kinematic chain of the mechanism is carried out. The kinematic chain is reduced from 6 DOF (artificial) to 2 DOF (planar) as shown in Figure 9.

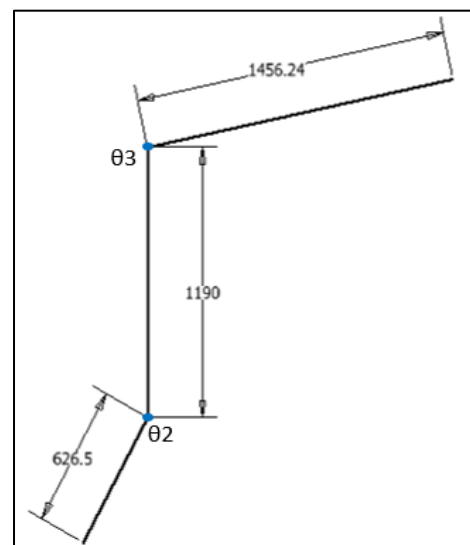


Figure 9. Planar kinematic chains

After forming a simpler kinematic chain, the DH parameters for the mechanism are shown in Table 3.

Table 3. DH Simple parameters

Join	a (mm)	d(mm)	$\theta$
0-1	1190	0	$\theta_2(90^\circ)$
1-2	1456,24	0	$\theta_3(281,85^\circ)$

From the calculations that have been carried out, it is found that the singular area is as shown in Figure 10. It can be seen that the manipulator has a singular configuration when  $\theta_3 = -78.15$ .

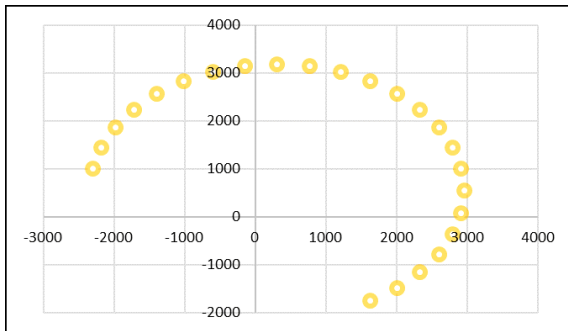


Figure 10. Singularity area

After doing 3D modeling for the manipulator to be tested, the dimensions of the manipulator in the default configuration are: height is 2197.35 mm, width 760 mm and length is 2140 mm.

In the forward kinematic calculation, there are differences in the results between calculations with Matlab and Inventor. This difference in results is caused by discrepancies that occur in the 3D assembly process, such as the incompatibility of the dimensions of the manipulator parts, the mismatch of the reference axes of each part so that the input angle value given cannot be precise.

The mechanism undergoes a singular configuration when 3 reaches  $-78.15$ , therefore when operating the input angle value given to 3 must be limited so that it does not touch that value.

#### 4. Conclusion

From the calculations that have been carried out, it can be concluded that:

1. The maximum reach that can be achieved by the manipulator is  $x = -2114,7$  mm to 2945.4 and  $y = -905.4$  mm to 3195.4 mm.
2. One of the manipulator positions that are close to the singular configuration is when 3 reaches  $-78.15$ .

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